



En raison du festival Balélec, nous avons dû procéder au(x) changement(s) de salle suivant pour votre cours :

<b>Matière</b>	<b>Enseignant</b>	<b>Classe</b>	<b>Date</b>	<b>Heure</b>	<b>Salles concernées</b>	<b>Nouvelle salles</b>
PHYS-510	Pivetta/Rusponi	PH-MA2	08.05.2025	8h-12h	CM 1 221	PH H3 31



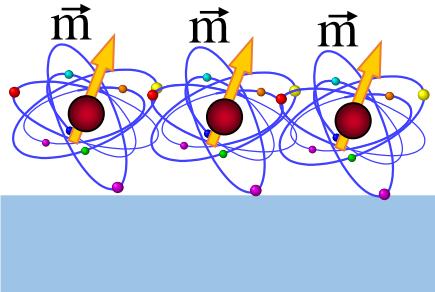
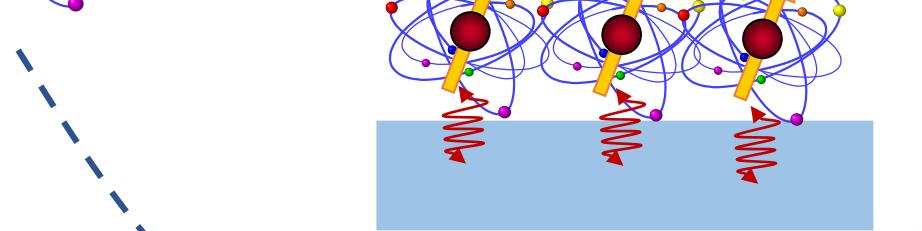
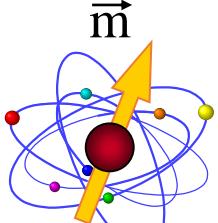
## *Lecture 7*

### *Spin Orbit Torque (SOT)*

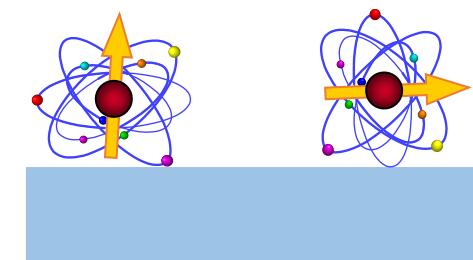


# The spintronics “goose game”

## Atom magnetism

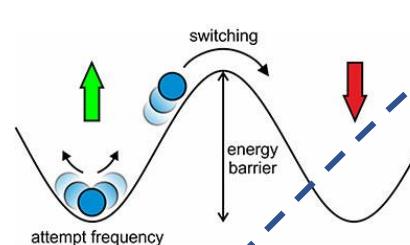


magnetic moment in a cluster and/or on a support

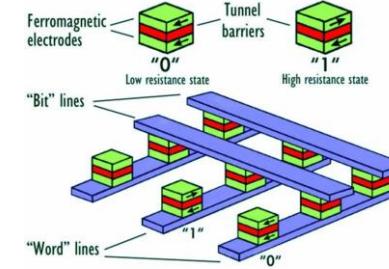
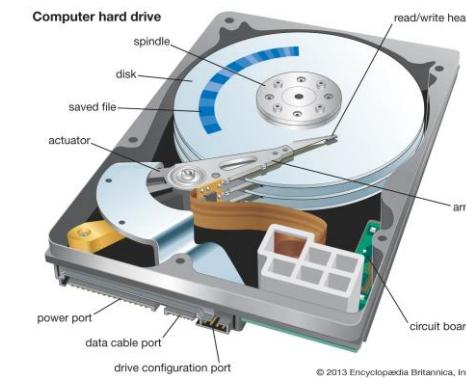


Magnetization easy axis

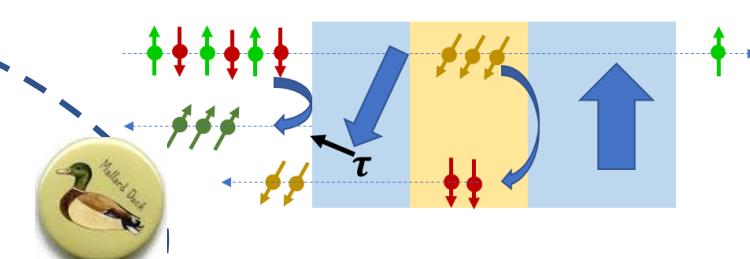
interactions between spins and with the supporting substrate



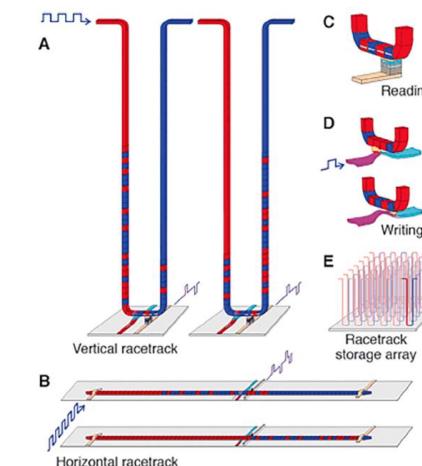
## applications



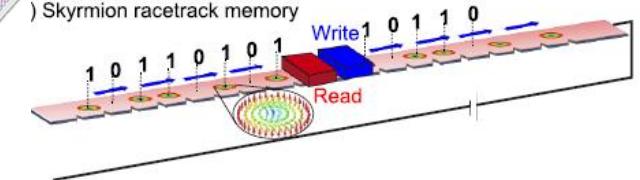
## STT - SOT

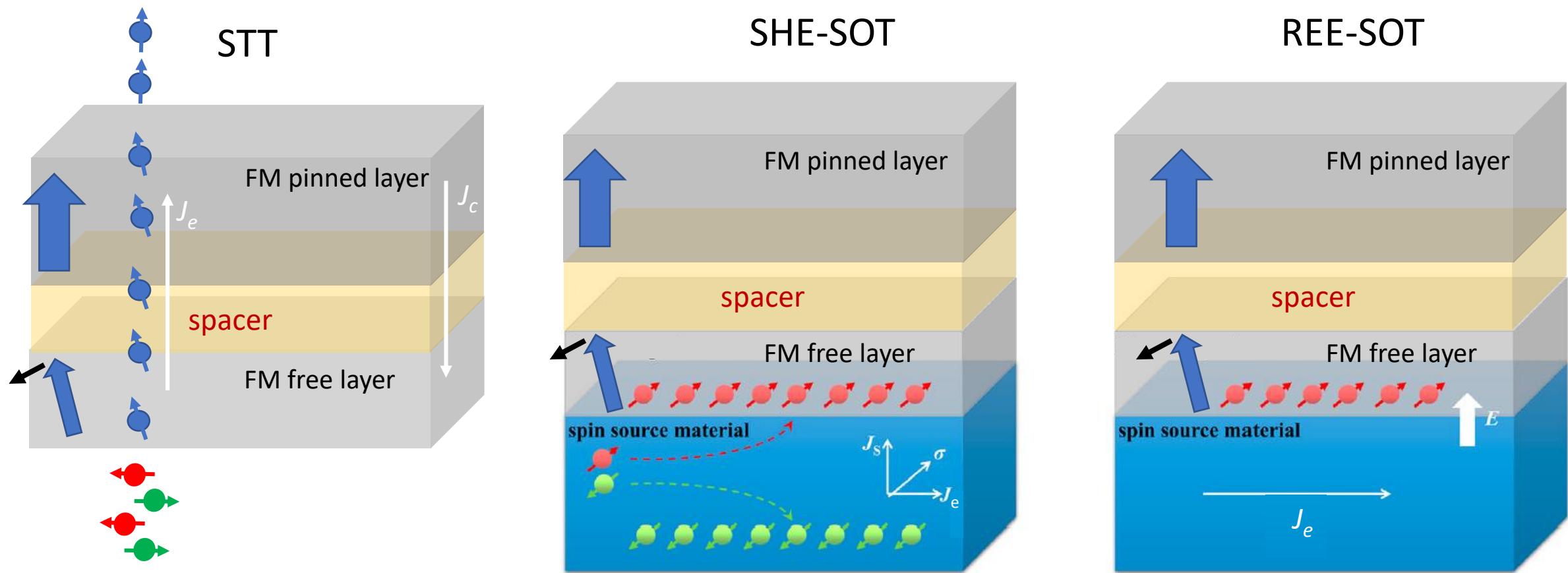


## Future



Skymion racetrack memory





SOT employs in-plane current injection to write:

- 1) separates the reading and writing paths
- 2) Less power required since current does not need to pass through the insulating spacer



A free electron with spin  $\sigma$  and moment  $\mathbf{p}$  moving in an external magnetic field  $\mathbf{B}$  feels:



$$\mathbf{F}_{Lorentz} = \frac{-e}{m} \mathbf{p} \times \mathbf{B}$$

$$H_{Zeeman} = -\mu_B \mathbf{B} \cdot \sigma$$

In crystals, electrons move with relativistic speed in the gradient of the crystal field potential. In the electron rest frame, this correspond to a magnetic field:



$$\mathbf{B}_{eff} = \frac{1}{mc^2} \nabla V \times \mathbf{p}$$

Spin-orbit coupling:  $H_{SO} = -\mu_B \mathbf{B}_{eff} \cdot \sigma = -\mu_B / mc^2 (\nabla V \times \mathbf{p}) \cdot \sigma \propto \mathbf{l} \cdot \sigma$

A twofold degenerate spin level might be split by SOC into two levels with spin parallel ( $H_{SO} \propto \mathbf{l} \cdot \sigma$ ) and antiparallel ( $H_{SO} \propto -\mathbf{l} \cdot \sigma$ ) to the orbit.

However, such splitting is symmetry forbidden for central symmetric systems (impossible to define the direction for spin up and down).

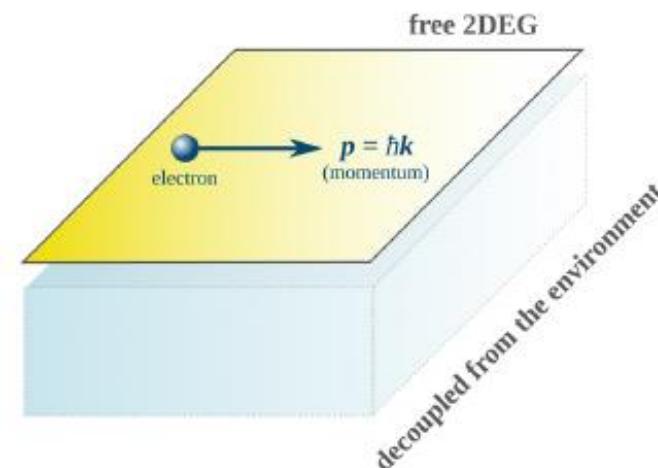


# Rashba-Edelstein effect (REE): 2D interfaces

The absence of an inversion symmetry center at the crystal surface or at 2D interfaces breaks this symmetry

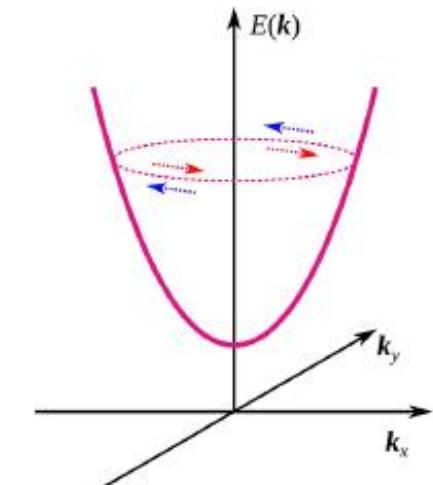
Free-standing 2D electron gas:  $V = V_{at}$   
Symmetric case

$$H = \frac{\mathbf{p}^2}{2m_e} + eV_{at} - \frac{e\hbar}{2m_e^2 c^2} (\nabla V_{at} \wedge \mathbf{p}_{//}) \cdot \boldsymbol{\sigma}$$



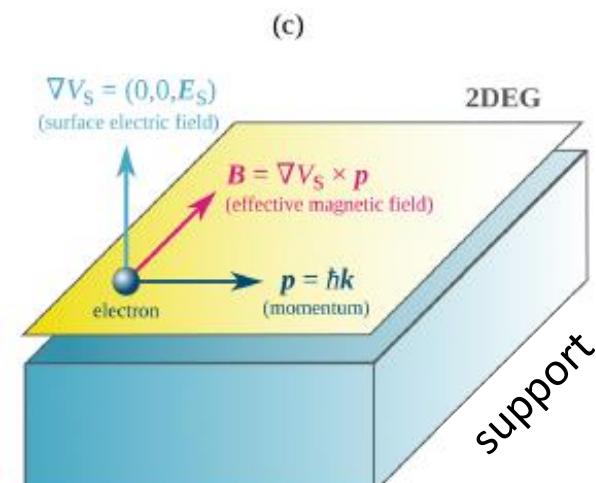
N.B.:

- 1) in these Hamiltonians  $V_{ee}$  is not included for simplicity
- 2) at surface electrons move parallel to the surface  $\rightarrow \mathbf{p} = \mathbf{p}_{//}$

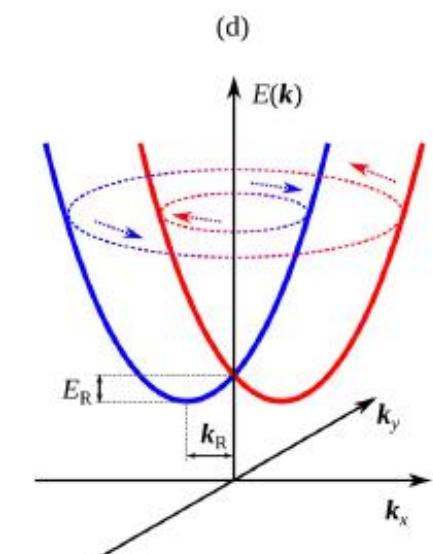


2D electron gas + support:  $V = V_{at} + E_s z$   
Broken symmetry

$$H = \frac{\mathbf{p}^2}{2m_e} + eV_{at} - \frac{e\hbar}{2m_e^2 c^2} (\nabla V_{at} \wedge \mathbf{p}_{//}) \cdot \boldsymbol{\sigma} + - \frac{e\hbar E_s}{2m_e^2 c^2} (\hat{\mathbf{z}} \wedge \mathbf{p}_{//}) \cdot \boldsymbol{\sigma}_{//} + eE_s z$$

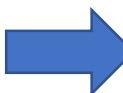


$E_s$  originates from the atomic potential perturbed by the breaking symmetry at interface





$$H = \frac{\mathbf{p}^2}{2m_e} + eV_{at} - \frac{e\hbar}{2m_e^2 c^2} (\nabla V_{at} \wedge \mathbf{p}_{//}) \cdot \boldsymbol{\sigma} +$$
$$- \frac{e\hbar E_s}{2m_e^2 c^2} (\hat{\mathbf{z}} \wedge \mathbf{p}_{//}) \cdot \boldsymbol{\sigma}_{//} + eE_s z$$

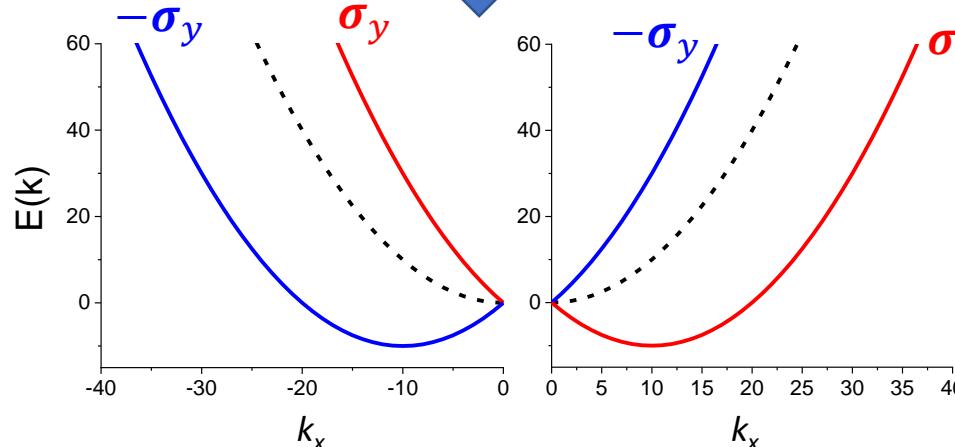


$$H = \frac{\mathbf{p}^2}{2m_e} + eV + V_{SO} \mathbf{l} \cdot \boldsymbol{\sigma} +$$
$$H_{RE} + eE_s z$$

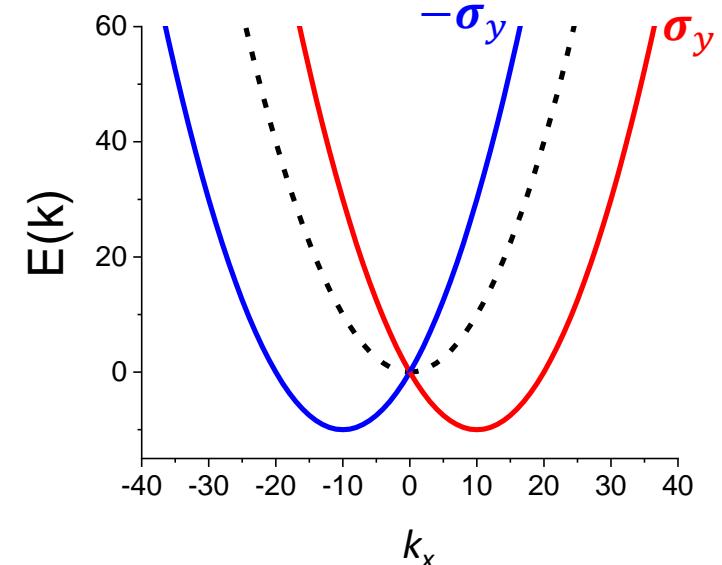
RE (Rashba-Edelstein) Hamiltonian:

$$H_{RE} = \frac{-e\hbar E_s}{2m_e^2 c^2} (\hat{\mathbf{z}} \wedge \mathbf{p}_{//}) \cdot \boldsymbol{\sigma}_{//} = \frac{-e\hbar^2 E_s}{2m_e^2 c^2} (\hat{\mathbf{z}} \wedge \mathbf{k}_{//}) \cdot \boldsymbol{\sigma}_{//} = \alpha_{RE} (\hat{\mathbf{z}} \wedge \mathbf{k}_{//}) \cdot \boldsymbol{\sigma}_{//}$$

Inverted splitting between spin polarized bands at inverted wave vector  $\mathbf{k}$  (black corresponds to  $H_{RE} = 0$ )



1D case



Because of SOC:

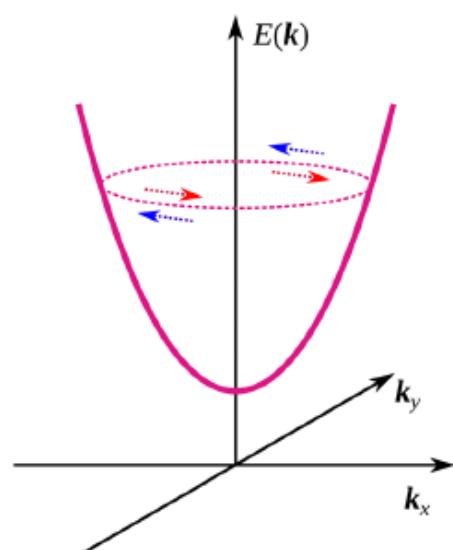
- 1) spins align perpendicularly to the momenta
- 2) splitting of spin bands



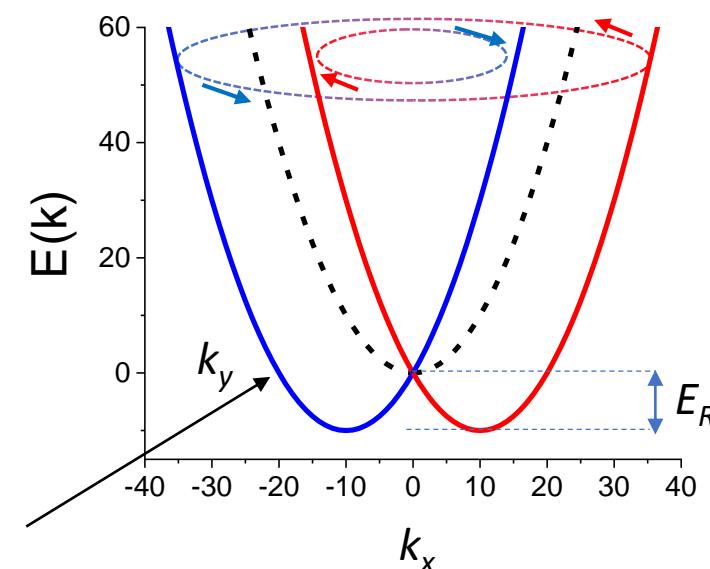
Because of SOC:

- 1) spins align perpendicularly to the momenta
- 2) splitting of spin bands

$$H_{RE} = 0$$

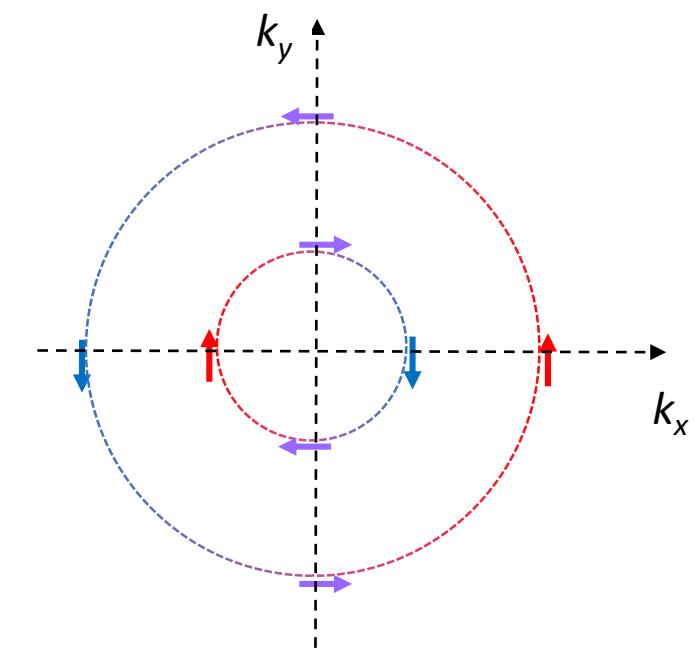


Chiral spin texture characterized by  $\mathbf{k}_x\sigma_y$  ( $\mathbf{k}_y\sigma_x$ ) spin-momentum locking



$E_R$  is used to quantify the REE

Top view

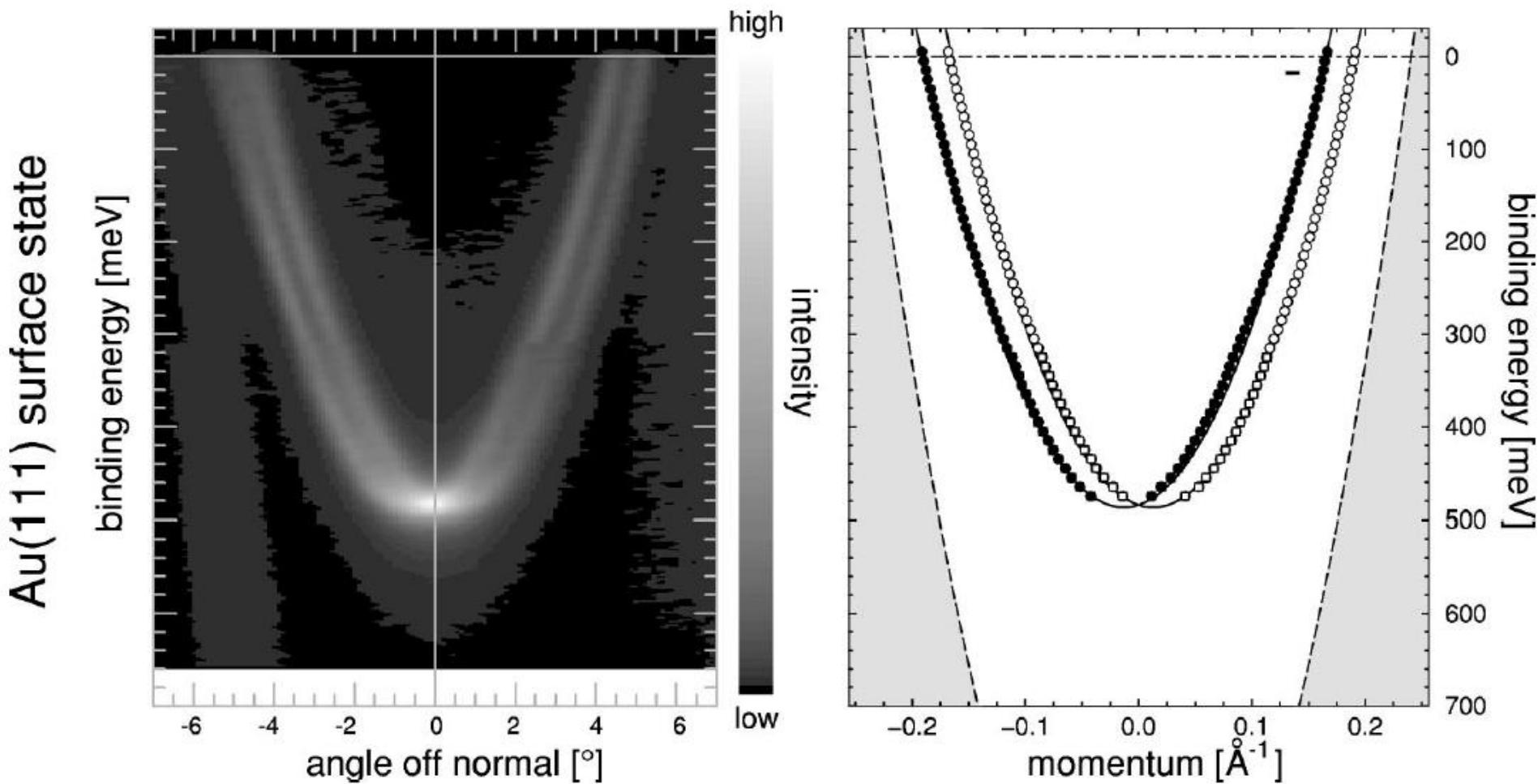


N.B.: arrow color code reflects the  $\sigma_y$  character



# Rashba splitting of Au(111) surface state

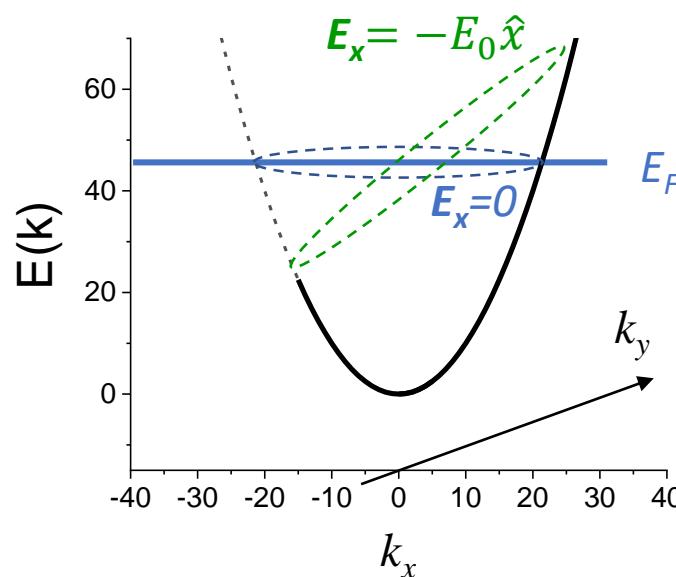
See exercises: 7.1-7.2



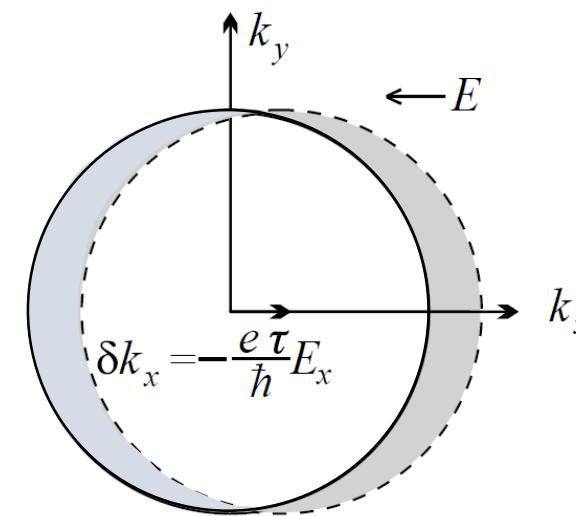


# Free standing 2D electron gas in an applied electric field

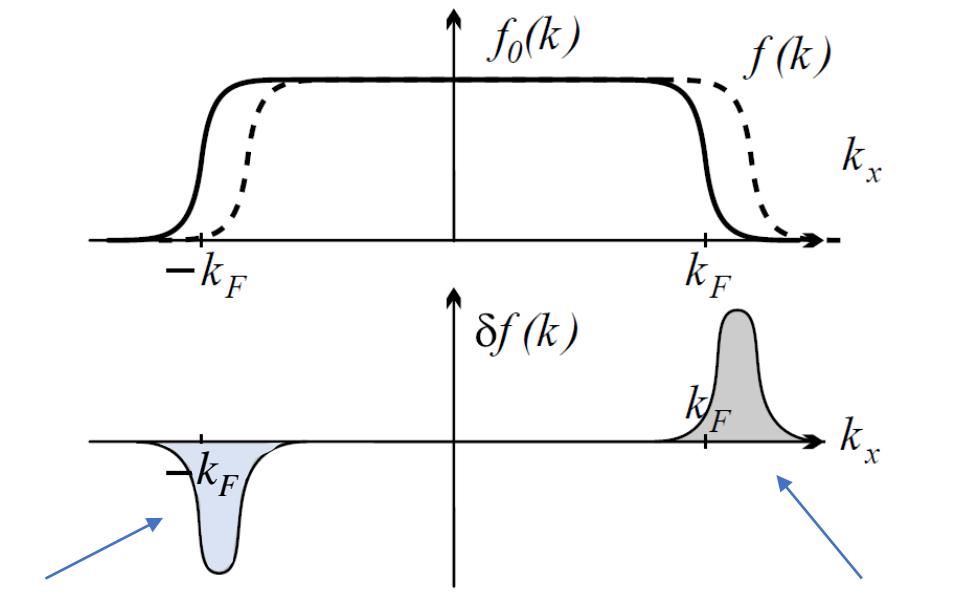
Band dispersion



Momentum space



Emptied states



Extra filled states

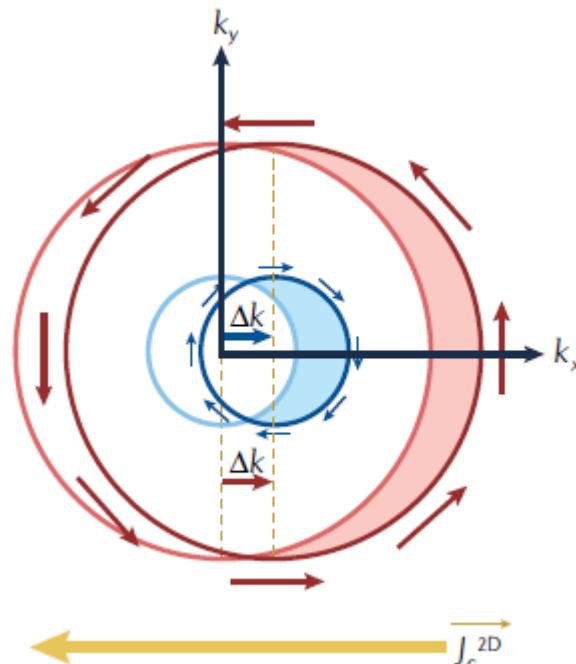
Displacement of the Fermi surface under the effect of an electric field generating an unbalanced distribution of states in the  $k$ -space:  
increased (decreased) number of states with positive (negative)  $k_x$  moment

N.B.: in the sketch the blue-gray (gray) part shows the states emptied (filled) as a consequence of the applied electric field



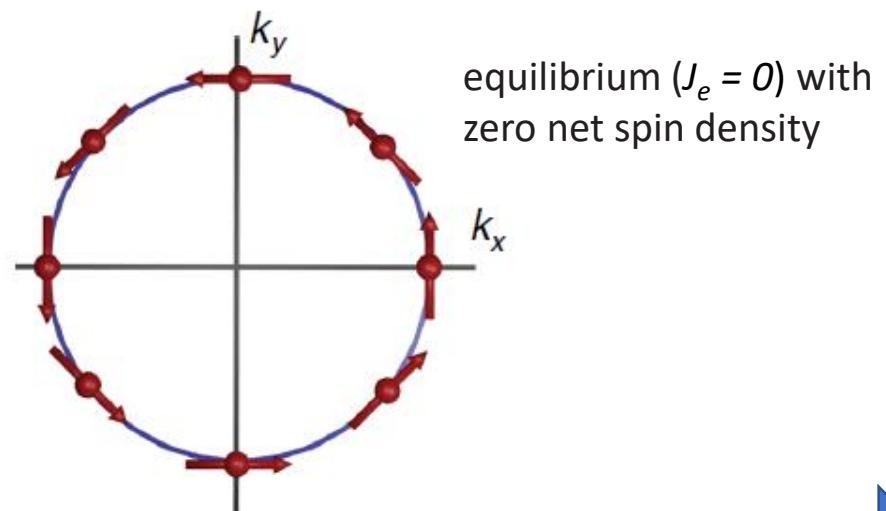
Rashba spin texture for the majority chiral states  
The reversed chirality will give an opposite but lower contribution

Displacement of the Fermi surface  
under the effect of an electric field

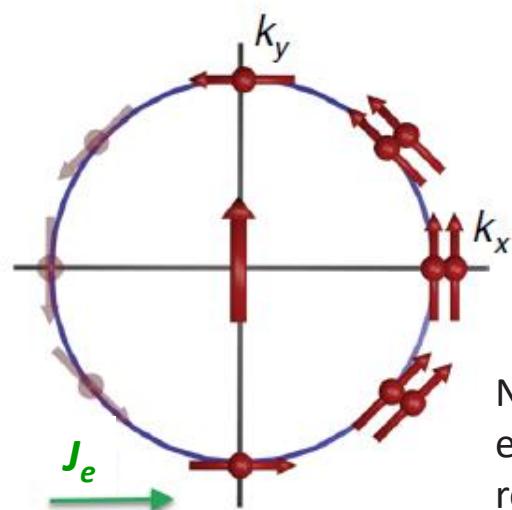


Current flows opposite to electrons

$$\mathbf{J}_c = -\mathbf{J}_e$$



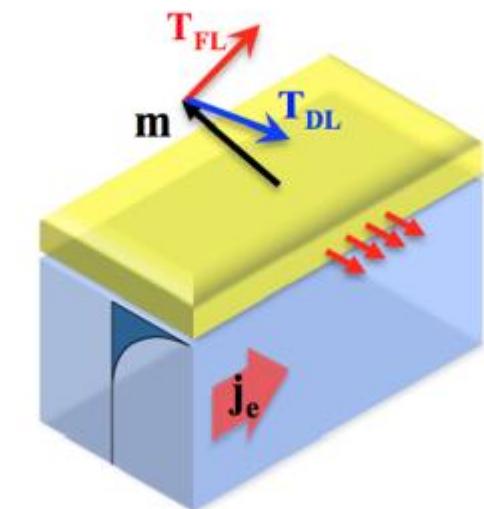
equilibrium ( $J_e = 0$ ) with  
zero net spin density



Nonequilibrium redistribution of  
eigenstates in an applied electric field  
resulting in a nonzero spin density



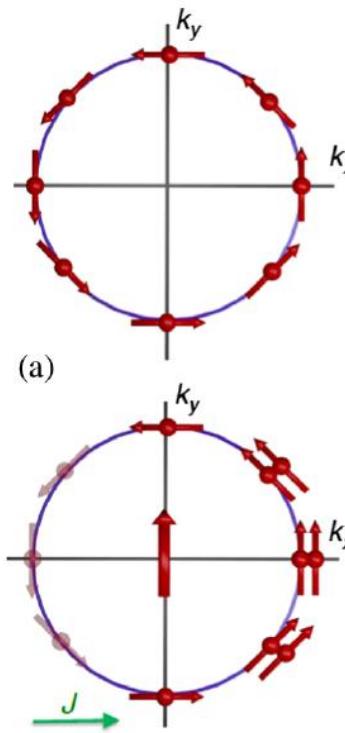
REE - SOT



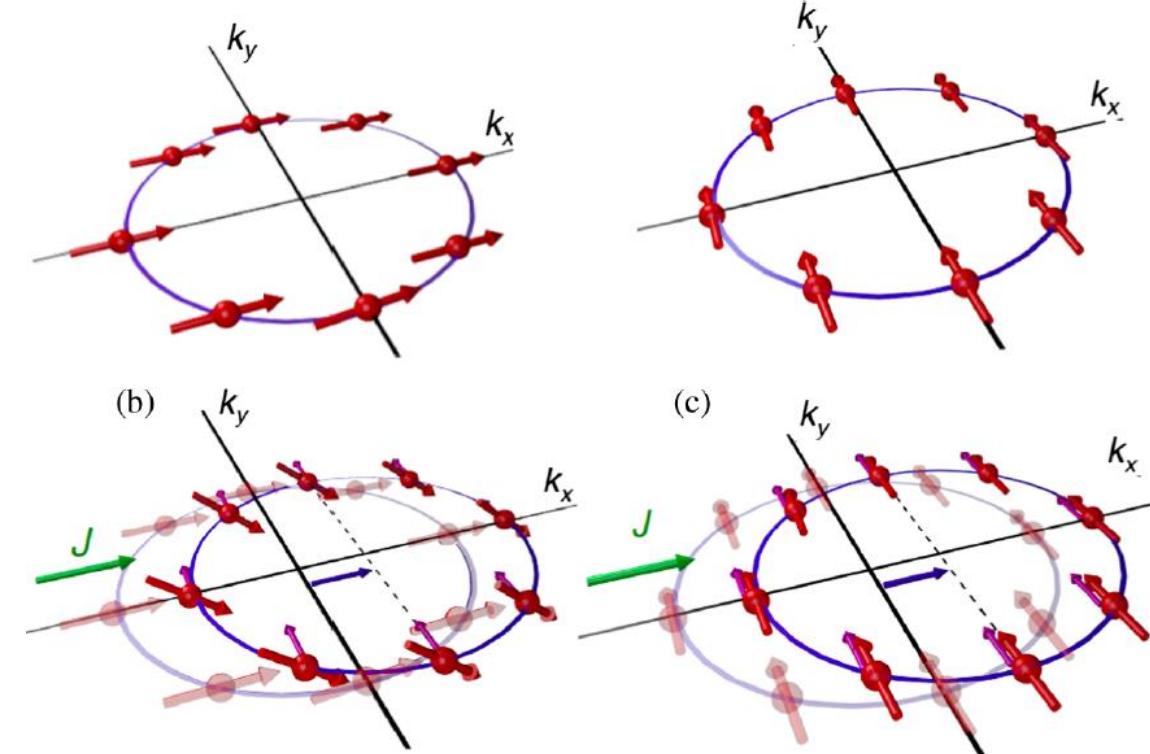
Torque on the top magnetic  
layer via exchange



## Non magnetic material



## Ferromagnetic material



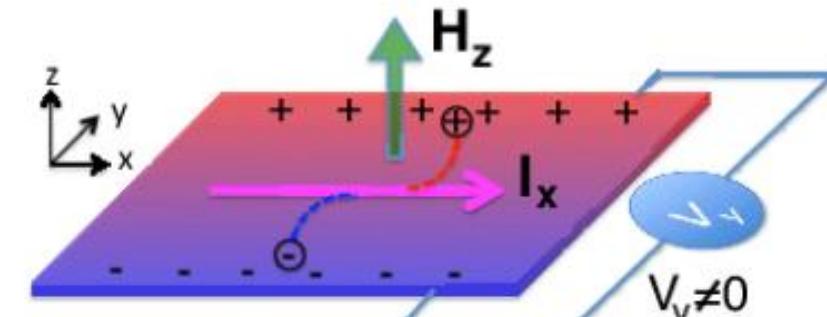
**(a) Top:** Rashba spin texture for one of the chiral states in equilibrium with zero net spin density. **Bottom:** Nonequilibrium redistribution of eigenstates in an applied electric field resulting in a nonzero spin density due to broken inversion symmetry of the spin texture. The exchange coupling of the carrier spin density to magnetization overlayer is responsible for the REE-SOT torque

**(b) Top:** A model equilibrium spin texture in a 2D Rashba spin-orbit coupled system with an additional time-reversal symmetry breaking exchange field (magnetization) of a strength much larger than the spin-orbit field. In equilibrium, all spins in this case align approximately with the  $x$  direction of the exchange field (magnetization). **Bottom:** In the presence of an electrical current along the  $x$  direction the Fermi surface (circle) is displaced along the same direction. When moving in momentum space, electrons experience an additional spin-orbit field proportional to  $\hat{z} \wedge \mathbf{p}_x$  (purple arrows). In reaction to this nonequilibrium current-induced field ( $\frac{dm}{dt} = -\gamma(\mathbf{m} \wedge \mathbf{H}_{eff})$ ), spins tilt and generate a uniform, nonequilibrium out-of-plane spin density (REE-SOT torque).

**(c) Top:** Same as in (b) for the  $y$  direction of the exchange field. **Bottom:** Same as in (b) but now with the current-induced spin-orbit field align with the exchange field, resulting in zero tilt of the carrier spins

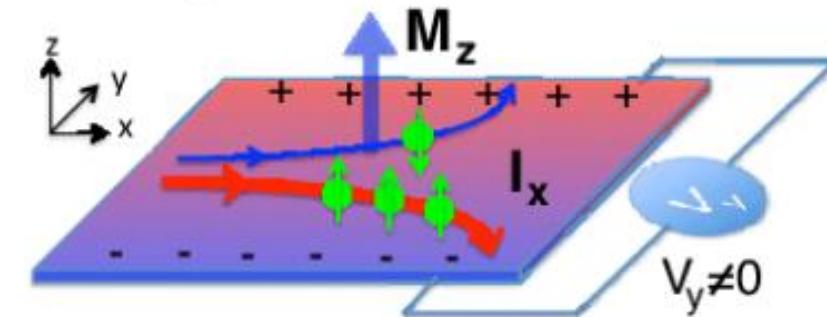


Hall effect (HE): The longitudinal current  $I_x$  under vertical external magnetic field  $H_z$  contributes to the transversal voltage  $V_y$  due to the Lorentz force experienced by carriers.



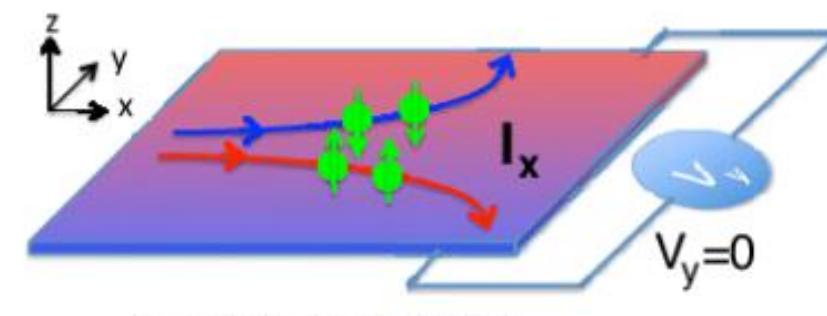
(a) Hall effect

Anomalous Hall effect (AHE): The electrons with majority and minority spin (due to spontaneous magnetization  $M_z$ ) have opposite "anomalous velocity" due to spin-orbit coupling. The spin polarization of the current causes unbalanced electron concentration at two transversal sides and leads to finite voltage  $V_y$ .



(c) Anomalous Hall effect

Spin Hall effect (SHE): In nonmagnetic conductor, equivalent currents in both spin channels with opposite velocity leads to a net spin current in transversal direction (but with balanced electron concentration at both sides  $\Rightarrow V_y = 0$ )



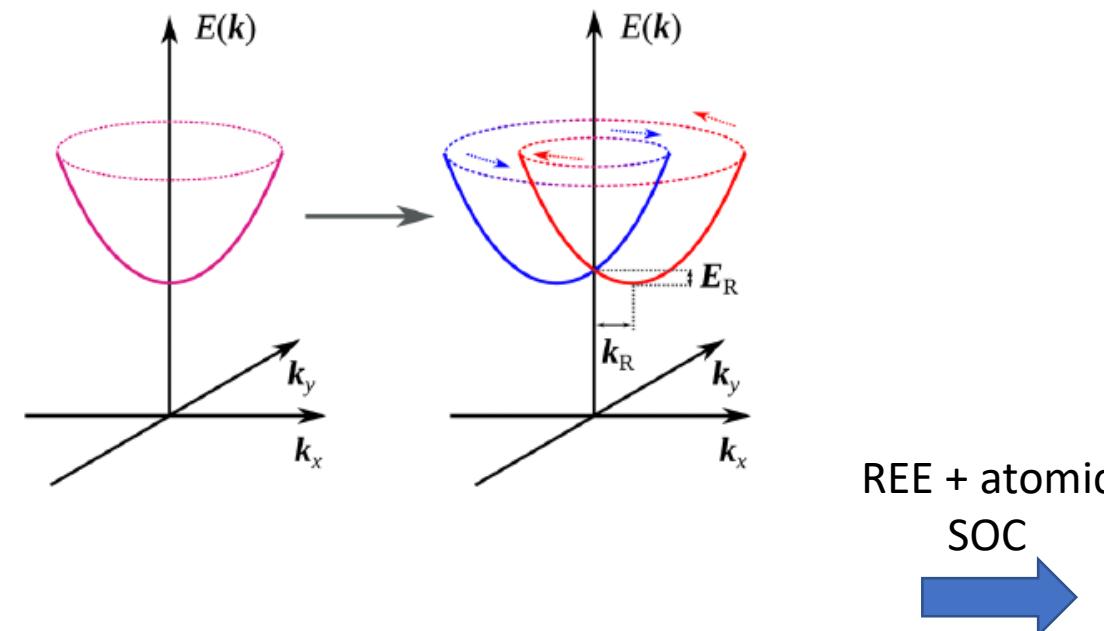
(e) Spin Hall effect



# Rashba vs. atomic SOC splitting

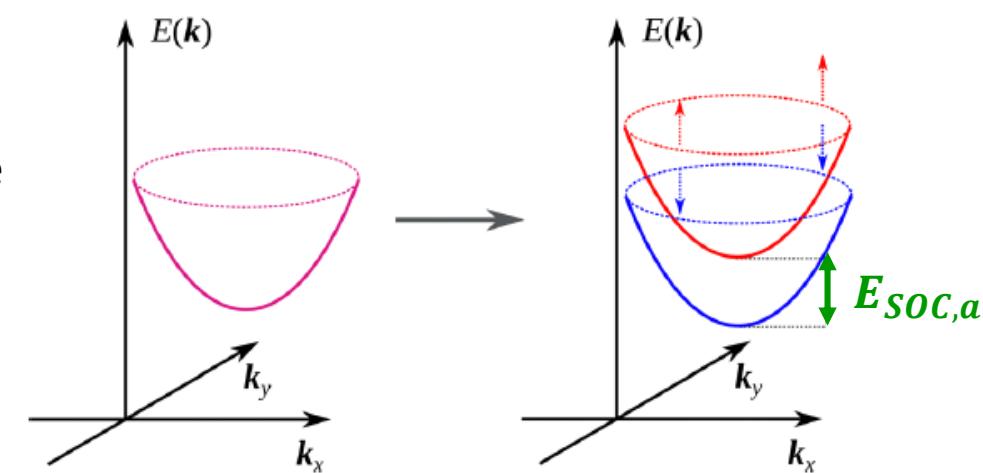
REE SOC: acts only on in-plane spins

$$E_R = \alpha_R (\hat{z} \wedge \mathbf{p}_{//}) \cdot \boldsymbol{\sigma}_{//}$$

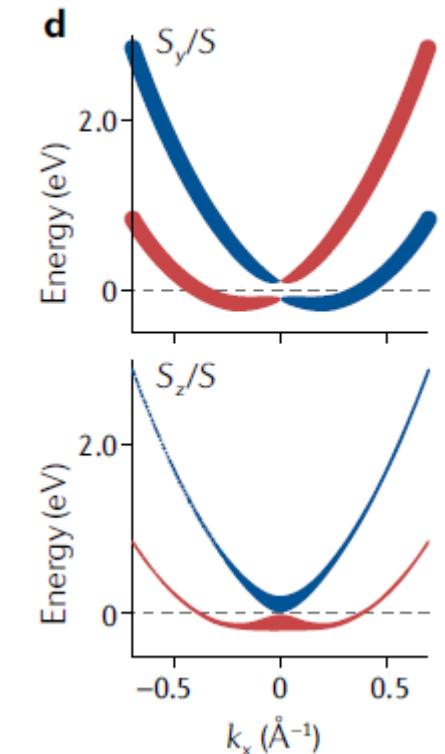


Atomic SOC: acts on both in-plane and out-of-plane spins

$$E_{SOC,at} = \frac{e\hbar}{2m_e^2c^2} (\nabla V_{at} \wedge \mathbf{p}) \cdot \boldsymbol{\sigma} = \\ = \mathbf{B}_{SOC,at} \cdot \boldsymbol{\sigma} \propto V_{SO} \mathbf{l} \cdot \boldsymbol{\sigma}$$



Magnetization along **z**



normalized  $S_x$  ( $S_y$ ) components as line thickness (different orientations are in red and blue). Reduced momentum locking for  $k \approx 0 \Rightarrow$  stronger  $S_z$  character



Spin split bands due to REE + Atomic SOC

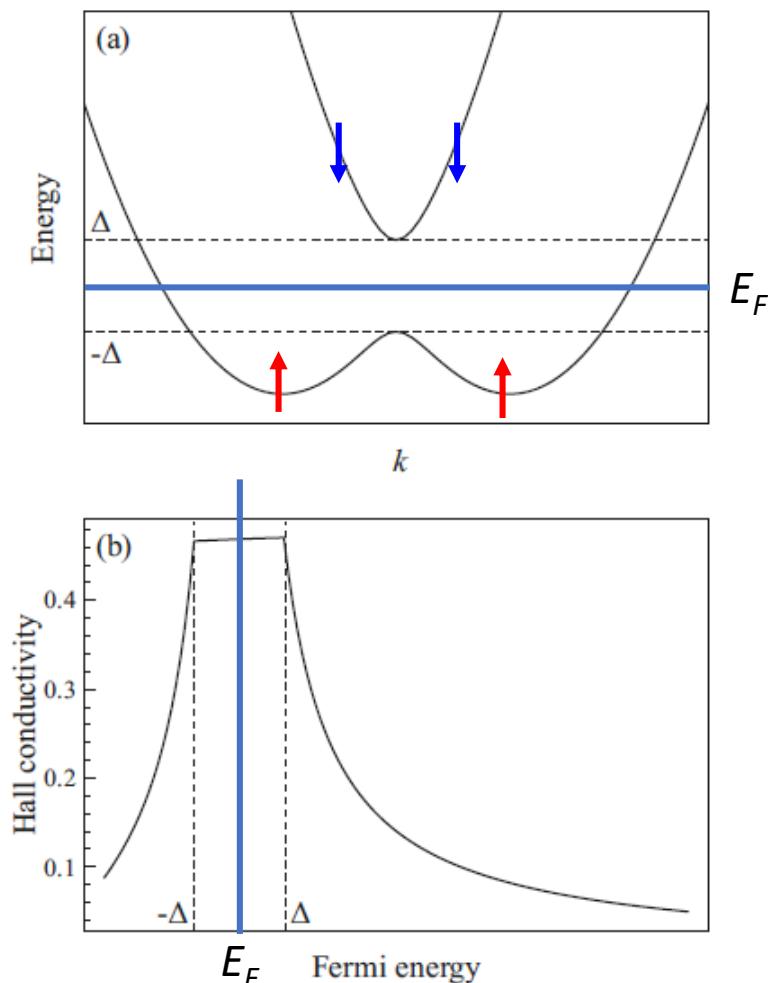


FIG. 6. Anomalous Hall effect in a simple two-band model. (a) Energy dispersion of spin-split bands. (b) The Hall conductivity  $-\sigma_{xy}$  in the units of  $e^2/h$  as a function of Fermi energy.

If  $E_F < -\Delta$ , the states with energies just below  $-\Delta$ , which contribute most to the AHE, are empty.

If  $E_F > \Delta$ , contributions from upper and lower bands cancel each other, and the AHE decreases quickly as  $E_F$  moves away from the band gap.

If  $E_F$  is in the gap region  $-\Delta < E_F < \Delta$ , the AHE is resonantly enhanced and reaches its maximum value about  $-e^2/2h$ .

Similar arguments hold for the spin-split polarized bands of a bulk material



Skew (Mott) scattering: inhomogeneous atomic potential results in an inhomogeneous effective magnetic field

$$\text{Atomic potential: } V(r) = \frac{Ze}{4\pi\epsilon_0 r}$$

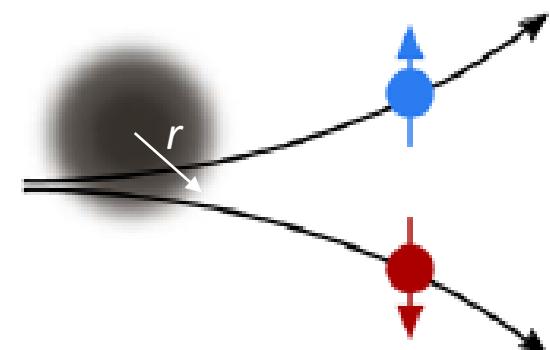
$$\nabla V = \frac{1}{r} \frac{dV(r)}{dr} \mathbf{r}$$

$$\mathbf{B}_{eff} = \frac{1}{mc^2} \nabla V \times \mathbf{p}$$

$\mathbf{B}_{eff}$  is a function of  $r \Rightarrow$  gradient of  $\mathbf{B}$

Similar to Stern-Gerlach experiment:  
opposite spins are deflected in opposite directions

Screw defects or impurities are the source of these extrinsic scattering events



AHE: unbalanced number of spin up and down  $\Rightarrow$

$$J_s \neq 0 \quad V_y \neq 0$$

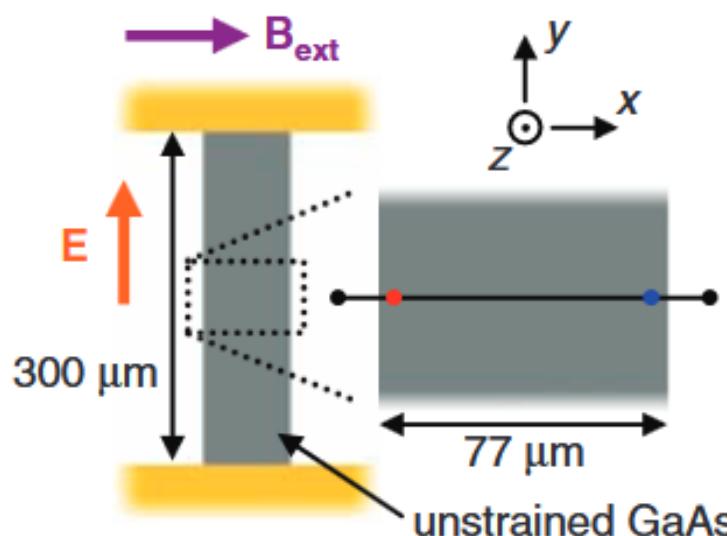
SHE: balanced number of spin up and down  $\Rightarrow$

$$J_s \neq 0 \quad V_y = 0$$



# Observation of SHE by magneto-optical Kerr effect

EPFL



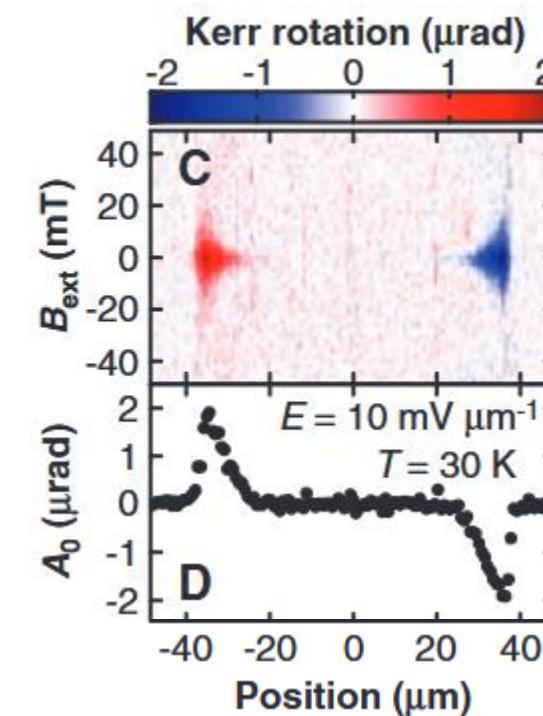
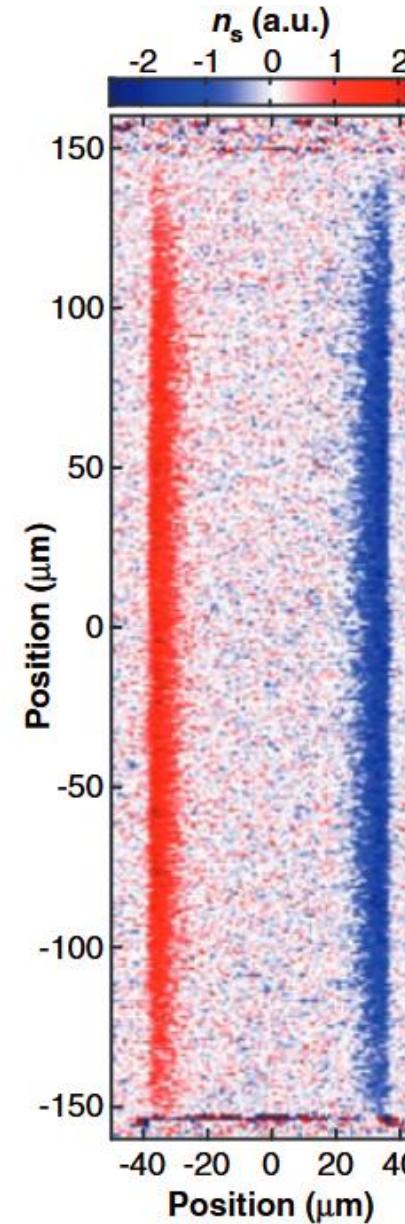
2  $\mu$ m thick GaAs

$n_s$  is the spin density

$T = 30$  K and  $E = 10$  mV/ $\mu$ m

$$J_s \approx 10 \text{ nA}/\mu\text{m}^2$$

$$J_c \approx 50 \text{ } \mu\text{A}/\mu\text{m}^2$$



Out-of-plane  
spin polarization

$$P_x = P_y = 0 \quad P_z(y) = P_z(0)e^{-y/L_s}$$

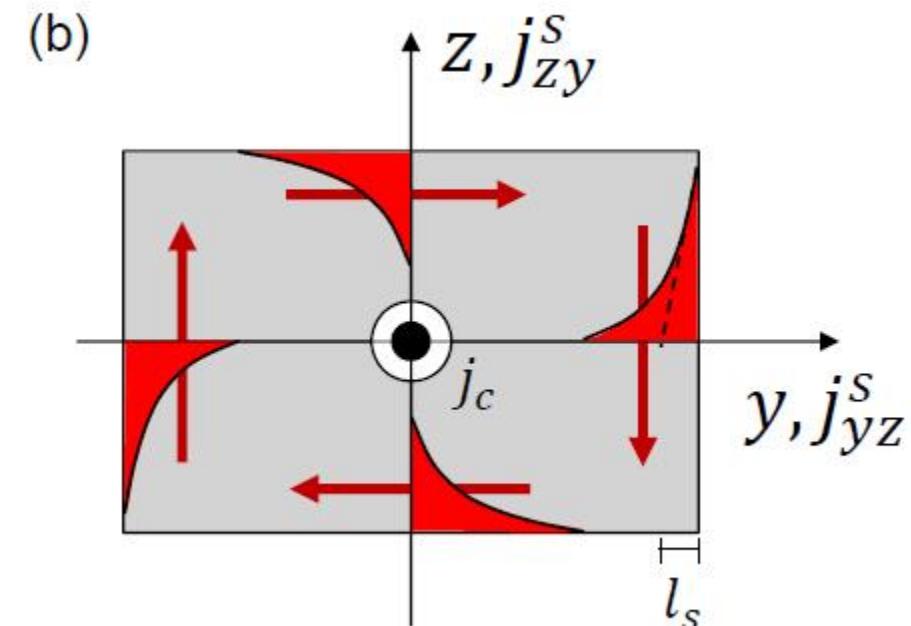
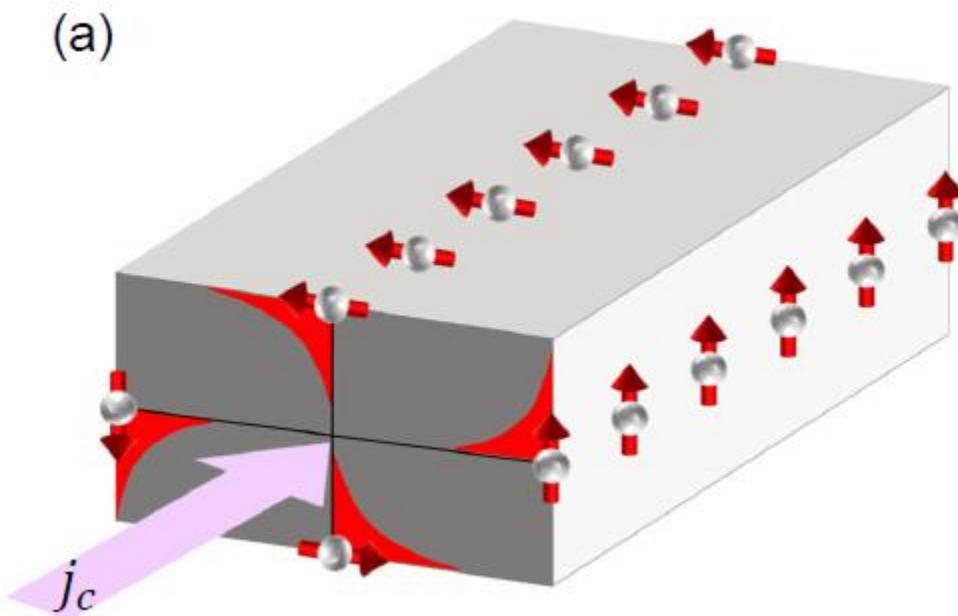
$$P_z(0) = -\frac{\gamma \mu n_s E L_s}{D}$$

$\gamma$  coefficient accounting for the SOC

$\mu$  and  $D$  are the mobility and the diffusion coefficient

$L_s = \sqrt{D\tau_s}$  is the spin diffusion length  $\approx 10 \mu\text{m}$

$\tau_s$  is the spin lifetime



Charge current:  $J_c = J^+ + J^-$

$$J_s = \theta_{SH} \frac{\hbar}{2e} J_c \wedge \sigma$$

Spin current:  $J_s = \frac{\hbar}{2e} (J^+ - J^-)$

$\theta_{SH}(Pt, Ta) \approx 0.05 - 0.2$   
spin to charge conversion factor



# Spin Hall effect: reversed spin polarized currents for Pt vs Ta

EPFL

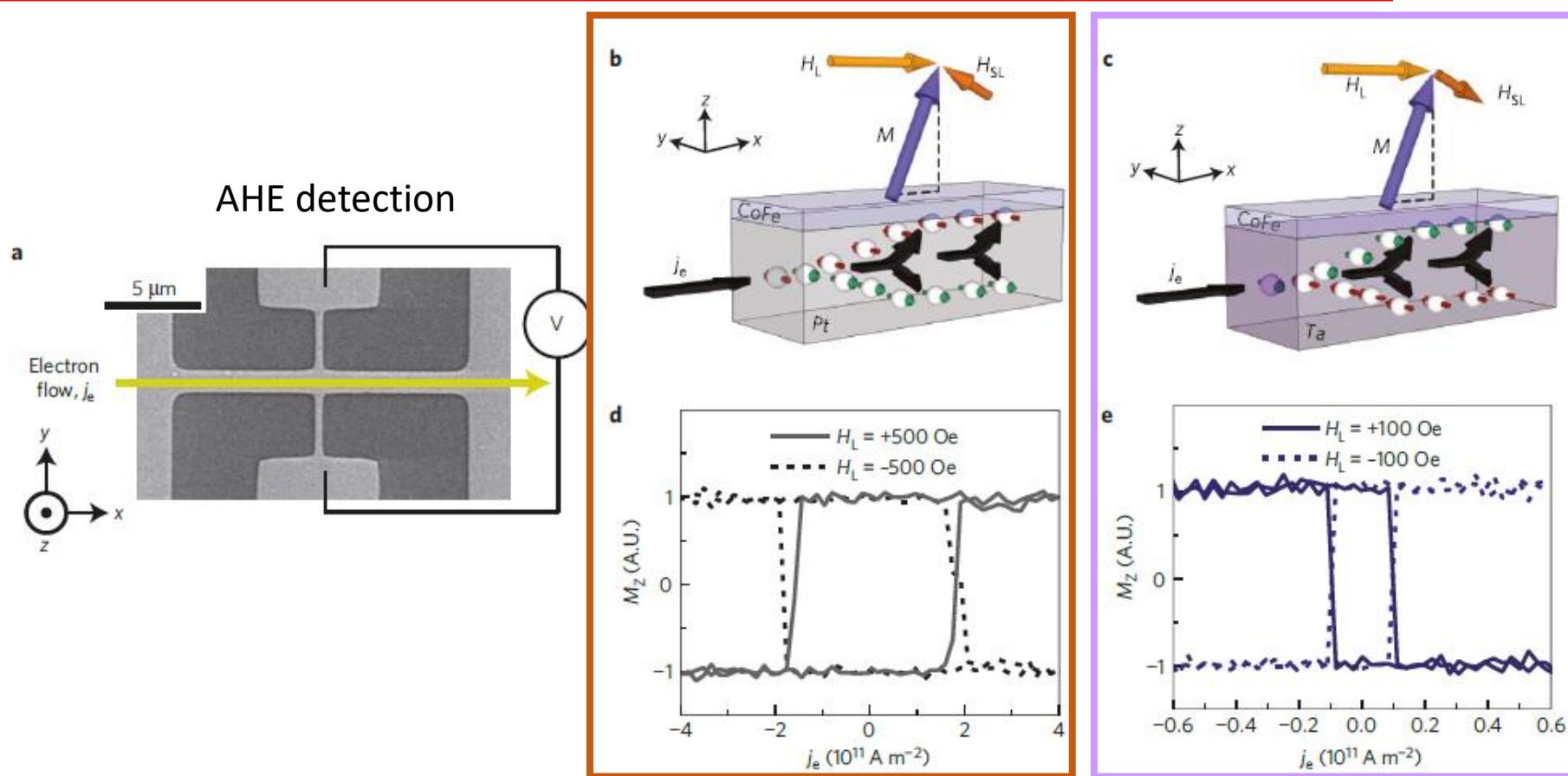
SOC has opposite effect

in Pt vs Ta

Pt: more than half filled

Ta: less than half filled

3	4	5	6	7	8	9	10	11	12
Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn
44.96	47.85	50.94	52.06	54.94	55.85	58.93	58.69	63.55	65.30
39	40	41	42	43	44	45	46	47	48
Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd
88.91	83.22	80.81	83.86	87.63	89.93	90.23	90.83	90.88	112.4
57-71	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg
134	172	183	185	187	188	189	190	191	205.5
89-103	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn
Actinides	Rutherfordium	Dubnium	Singapore	Bethes	Hassium	Magnesium	Darmstadtium	Bergeron	Copernicus

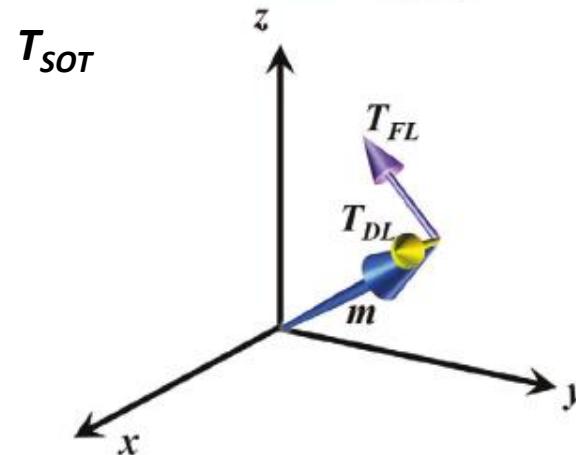
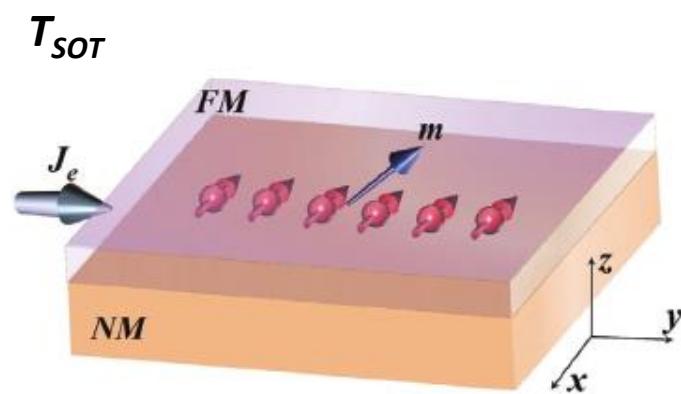
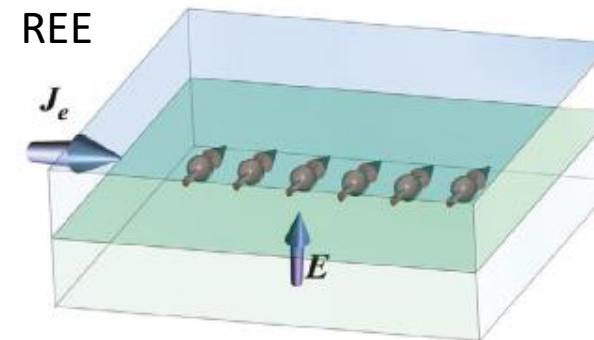
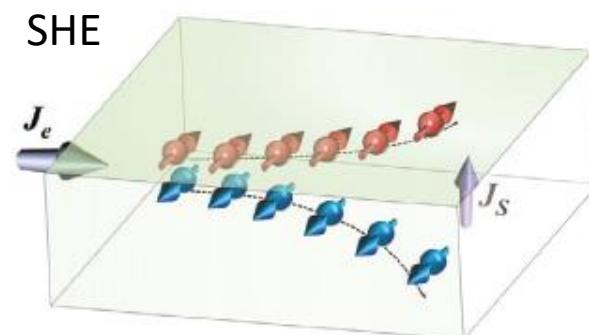


**Figure 2 | Current-induced switching under a constant in-plane longitudinal field.** **a**, Scanning electron micrograph of a Hall cross. **b,c**, Illustrations of Pt/CoFe/MgO (**b**) and Ta/CoFe/MgO (**c**) in the up magnetization state with the injected electron current and applied longitudinal field  $H_L$  in the  $+x$  direction. Owing to the combination of the current-induced Slonczewski-like torque (producing an effective field  $H_{SL}$ ) and the applied longitudinal field, up magnetization is stable in Pt/CoFe/MgO whereas it is unstable in Ta/CoFe/MgO. **d,e**, Out-of-plane magnetization  $M_z$  (normalized anomalous Hall signal) as a function of electron current density  $j_e$  under a constant  $H_L$  in Pt/CoFe/MgO (**d**) and Ta/CoFe/MgO (**e**). The magnitude of  $H_L$  is 500 Oe for Pt/CoFe/MgO (**d**) and 100 Oe for Ta/CoFe/MgO (**e**). When  $H_L$  is reversed from  $+x$  (solid line) to  $-x$  (dotted line), the stable magnetization direction under a given current polarity reverses.



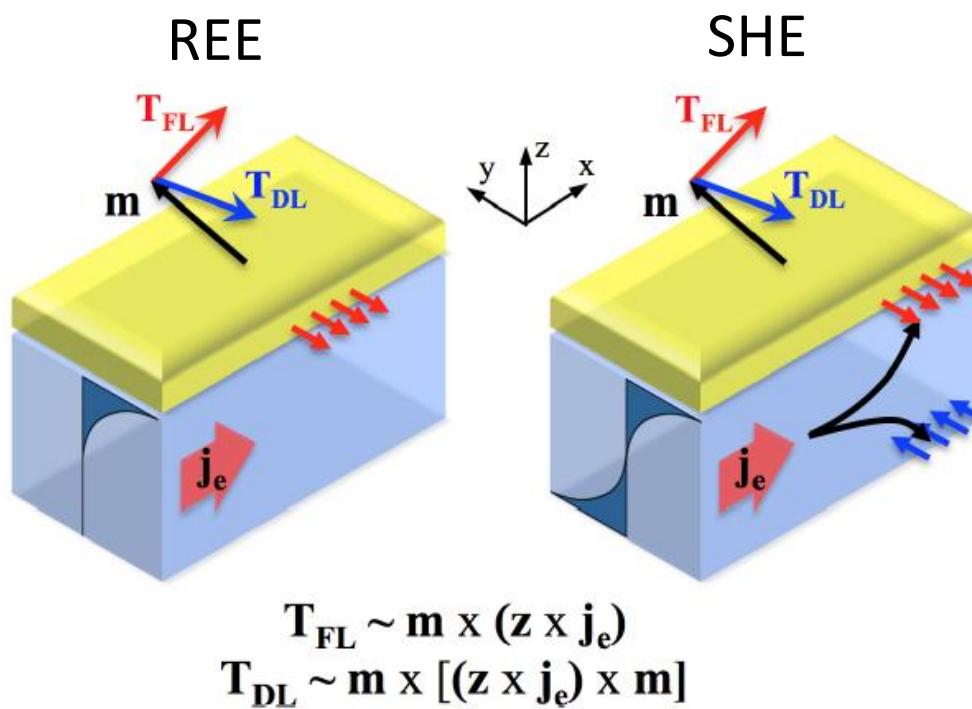
SHE: a charge current flowing in an NM layer generates a spin current owing to asymmetric spin deflection induced by SOC.

The polarization direction is perpendicular to both the directions of the charge and spin currents.

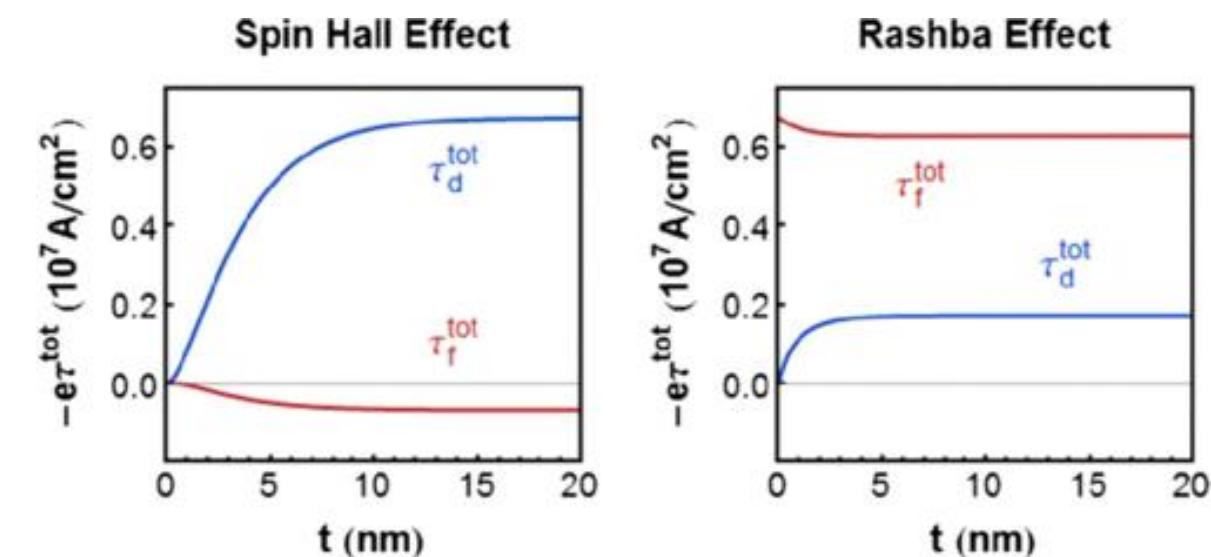


As the spin current diffuses into the adjacent FM layer, a SOT torque  $\mathbf{T}_{SOT}$  (via exchange) is exerted on the magnetization  $\mathbf{m}$ . This torque has two components  $\mathbf{T}_{SOT} = \mathbf{T}_{DL} + \mathbf{T}_{FL}$

REE: an internal electrical field  $E = E_z \mathbf{z}$  is generated at the interface/surface because of the spatial inversion symmetry breaking. When an in-plane charge current flows through the FM/NM heterostructure, the conduction electrons near the interface move in the electrical field  $E$ , and they experience an effective magnetic field perpendicular to the current direction (SOC-induced Rashba field  $\mathbf{H}_{RE}$ ): accumulation of spins perpendicular to both the charge current  $J_e$  and  $E = E_z \mathbf{z}$ .



Both mechanisms produce damping-like and field-like torques. The small red and blue arrows denote the nonequilibrium spin density accumulating at the interfaces, and their corresponding spatial distribution is sketched as a shaded area on the structure's side. The large red and blue arrows represent the field-like and damping-like torques, respectively.

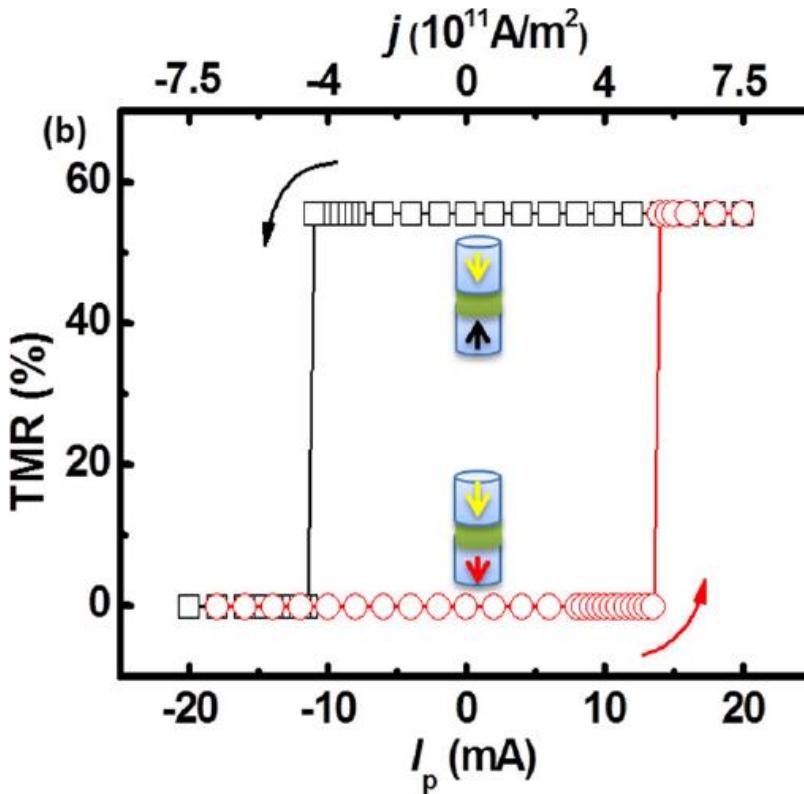


Torque components as a function of the nonmagnetic metal thickness



# SOT writing and TMR read-out in p-MTJ

EPFL



TMR as a function of current pulse amplitude

$I_p$  injected in the Ta electrode using 50 ns long pulses under an in-plane magnetic field  $H= -0.4$  kOe along the current. The arrows show the sweep direction of  $I_p$ .

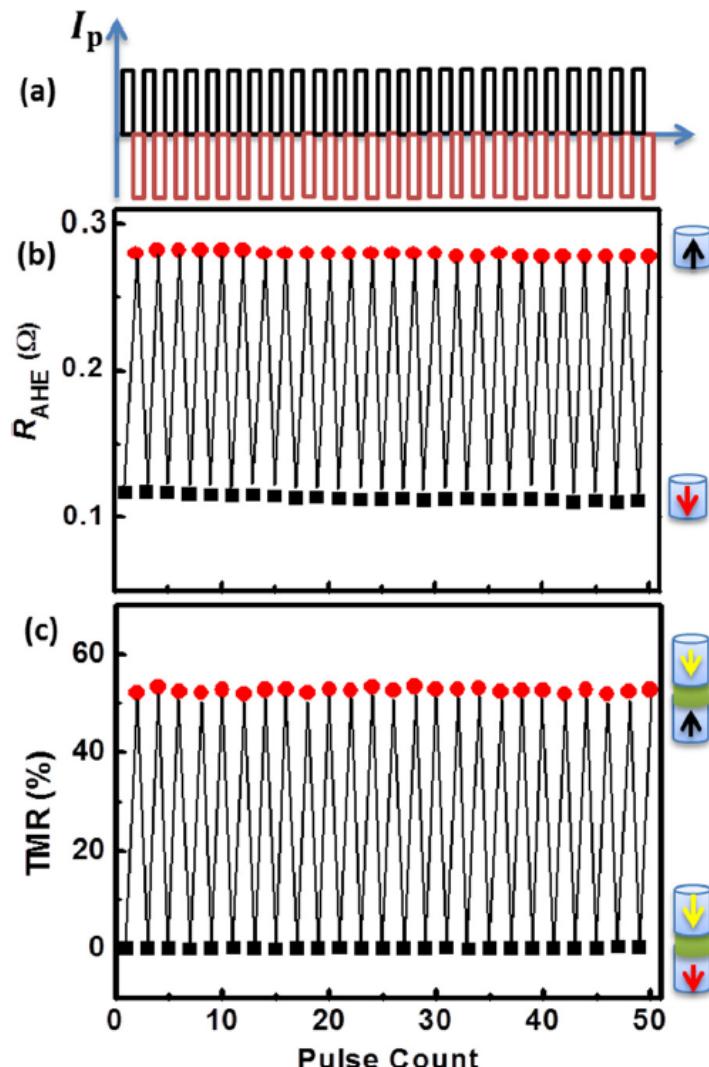
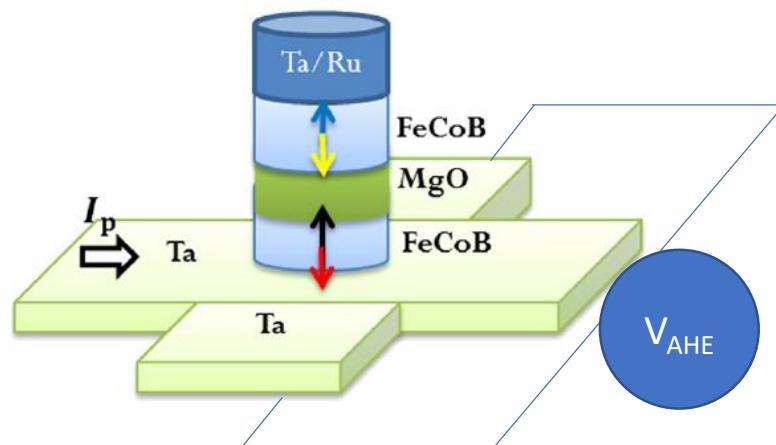
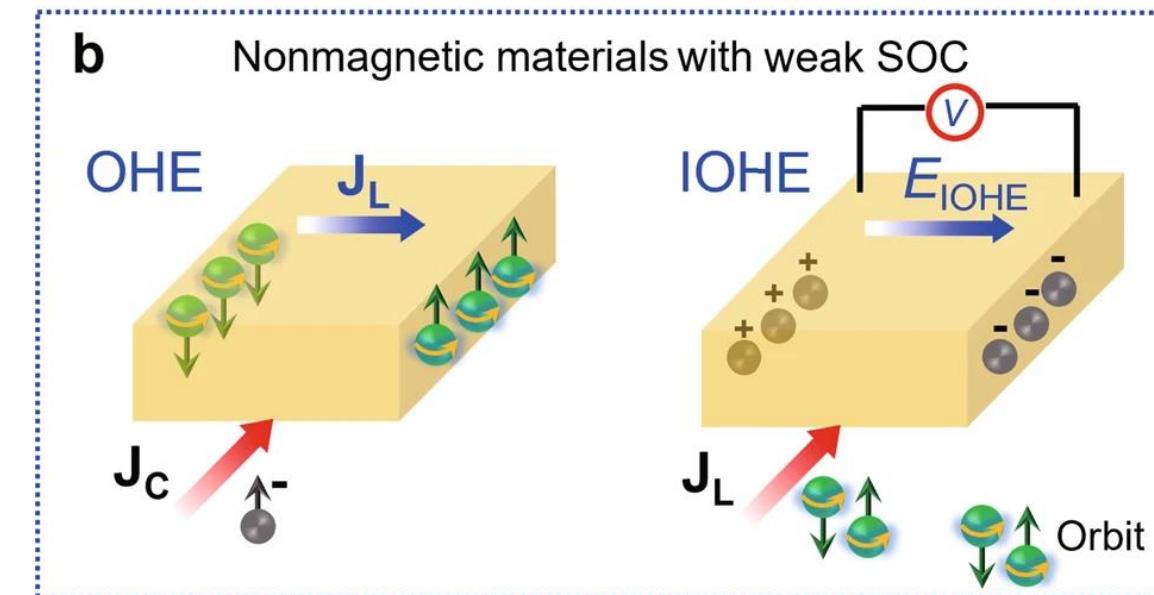
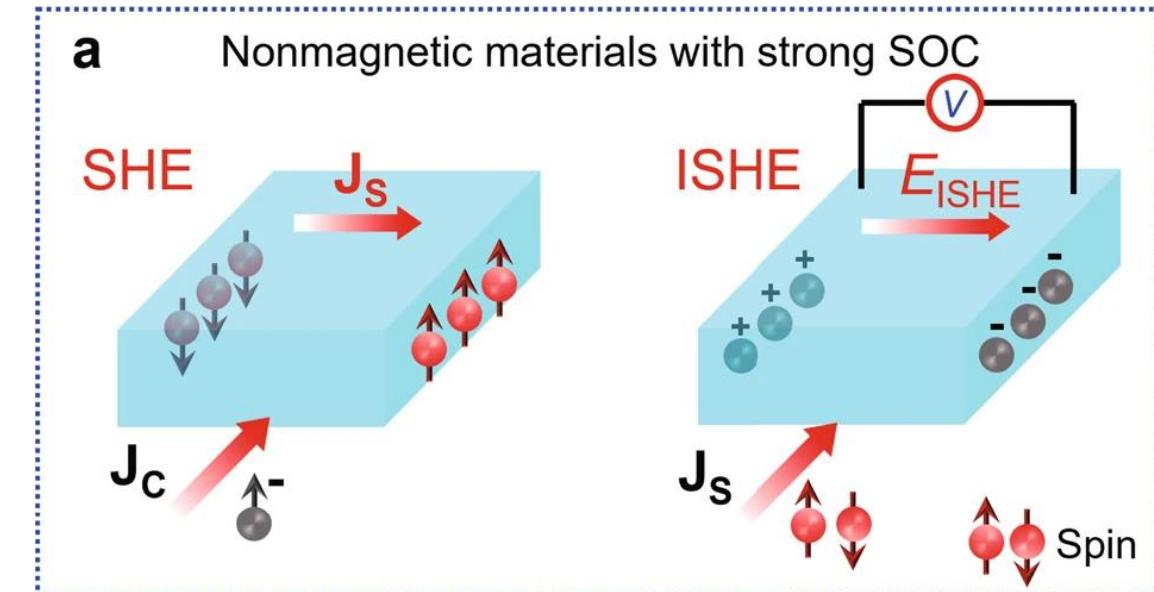


FIG. 3. (a) Schematic of the pulse sequence. (b) The AHE resistance (proportional to the  $M_z$  component of the bottom FeCoB layer) and (c) TMR measured after the injection of positive (black squares) and negative (red circles) current pulses of amplitude  $I_p = 20$  mA and 50 ns long under  $H_1 = -0.4$  kOe.



SHE and ISHE (invers SHE) refer to the conversions of  $J_c \rightarrow J_s$  and  $J_s \rightarrow J_c$  in the heavy metals with strong SOC, where a transverse flow of spin angular momentum and voltage are generated, respectively.

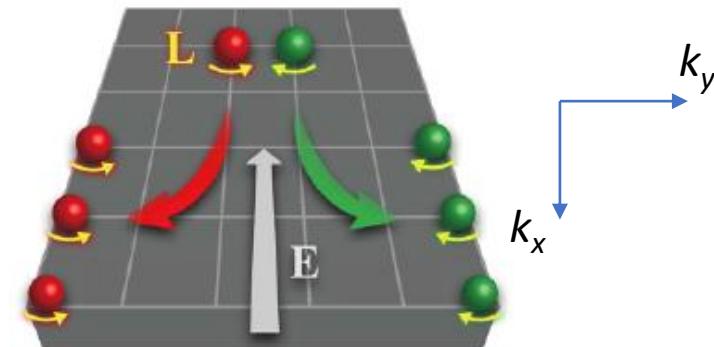


OHE and IOHE are the conversion of  $J_c \rightarrow J_L$  and  $J_L \rightarrow J_c$  in the materials with weak SOC, where a transverse flow of orbital angular momentum and voltage are induced, respectively. Thanks to SOC in an adjacent FM layer, the orbital current exerts a torque on the FM layer



## Schematic illustration of the OHE without SOC

The angular momentum  $\mathbf{L}$  is defined from localized orbitals around the atom at each lattice. In the presence of an external electric field  $\mathbf{E}$ , electrons with opposite  $\mathbf{L}$  deflect in the clockwise (red arrow) or anticlockwise (green arrow) direction.

Formal approach (ex. for  $p$ -waves)

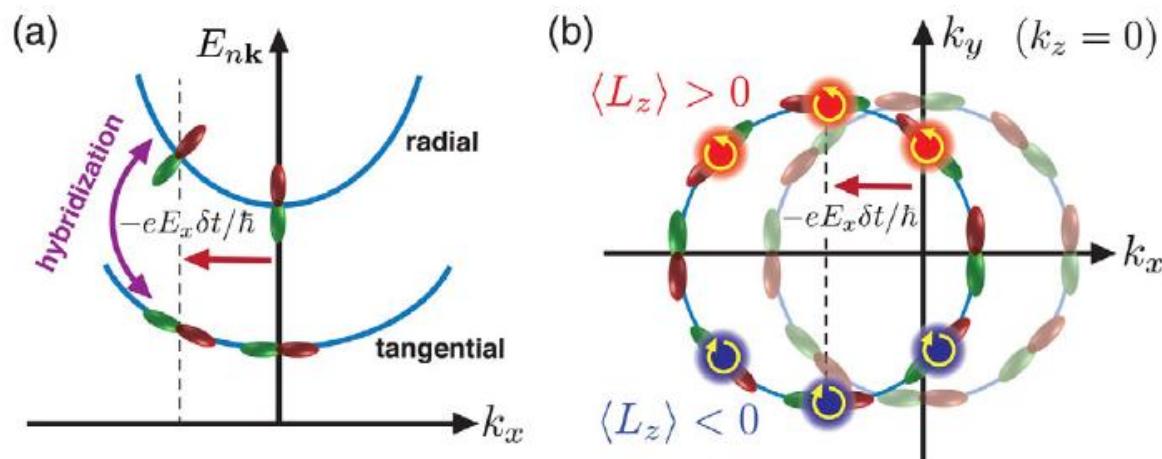
$$p_{radi} = \cos \phi p_x + \sin \phi p_y$$

$$p_{tang} = \sin \phi p_x - \cos \phi p_y$$

Under the effect of the external field,  $p_{radi}$  and  $p_{tang}$  get hybridized resulting in new states that can have finite  $L_z$

Table 2. Matrix elements  $\langle \mathbf{p}_i | \hat{\mathbf{e}} \cdot \mathbf{L} | \mathbf{p}_j \rangle$ .

	$\langle x  $	$\langle y  $	$\langle z  $
$ x\rangle$	0	$i\hat{e}_z$	$-i\hat{e}_y$
$ y\rangle$	$-i\hat{e}_z$	0	$i\hat{e}_x$
$ z\rangle$	$i\hat{e}_y$	$-i\hat{e}_x$	0

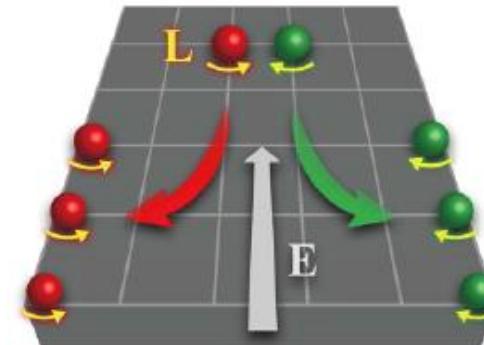


(a) Schematic band structure with plots of wave function character at each band. Here,  $k_y = k_z = 0$ .  
 (b) When an electron in the lower band is pushed from  $\mathbf{k}$  to  $\mathbf{k} + \delta\mathbf{k}$  by an external electric field  $\mathbf{E} = E_x \mathbf{x}$ , positive (negative)  $L_z$  is induced for the nonequilibrium state with  $k_y > 0$  ( $k_y < 0$ ).



## Schematic illustration of the OHE without SOC

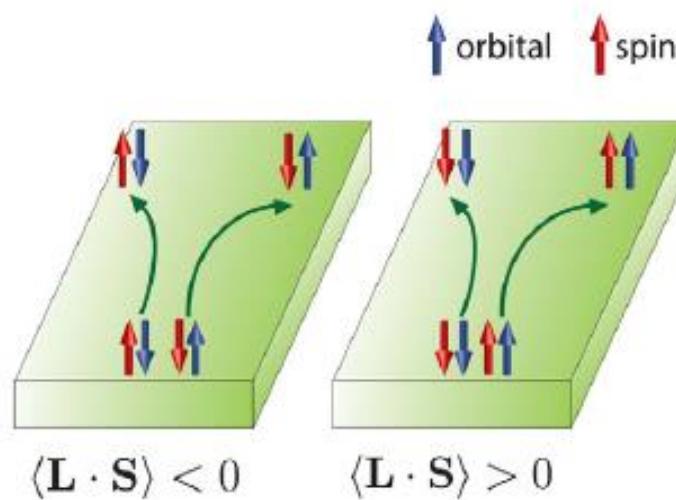
The angular momentum  $\mathbf{L}$  is defined from localized orbitals around the atom at each lattice. In the presence of an external electric field  $\mathbf{E}$ , electrons with opposite  $\mathbf{L}$  deflect in the clockwise (red arrow) or anticlockwise (green arrow) direction.



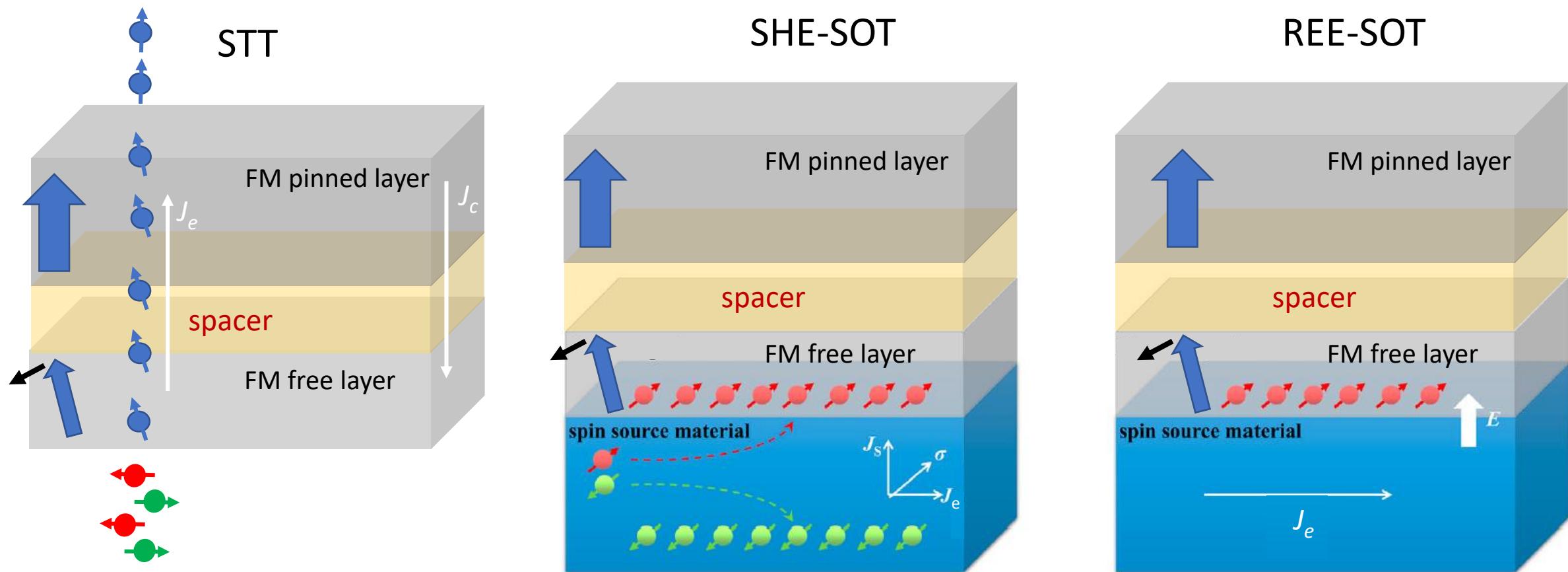
## Schematic illustration when SOC is taken into account.

Spin and orbital momenta are locked by SOC.

SHE occurs in the same or opposite direction of OHE depending on whether  $\langle \mathbf{L} \cdot \mathbf{S} \rangle$  is positive or negative.



The intrinsic SHE then emerges as a by-product of the OHE resulting from the orbital-to-spin conversion in materials with nonzero SOC



SOT employs in-plane current injection:

- 1) separates the reading and writing paths
- 2) Less power since current does not need to pass through the insulating spacer